Name ______________________ Date ______________

BENCHMARK 1
(Chapters 1 and 2)

A. Expressions, Equations, and Inequalities (pp. 1–3)

A **variable** is a letter used to represent one or more numbers. An **algebraic expression** is made from numbers, variables, and algebraic operations. The following examples describe how expressions can be evaluated, combined, written, and used to write algebraic equations and inequalities.

1. Evaluate Expressions

Vocabulary

Evaluate an expression Substitute a number for the variable, perform the operation(s), and simplify the result if necessary.

EXAMPLE

When a number is followed directly by a variable, the operation of multiplication is always implied.

Evaluate the expression.

a. $16n$ when $n = 4$ **b.** $\frac{25}{k}$ when $k = 5$ **c.** $h - 8$ when $h = 12.2$

d. $\frac{4}{3} + h$ when $h = \frac{1}{3}$ **e.** x^3 when $x = 4$ **f.** a^2 when $a = 1.2$

Solution:

a. $16n = 16 \cdot 4$
$= 64$

b. $\frac{25}{k} = \frac{25}{5}$
$= 5$

c. $h - 8 = 12.2 - 8$
$= 4.2$

d. $\frac{4}{3} + h = \frac{4}{3} + \frac{1}{3}$
$= \frac{5}{3}$

e. $x^3 = 4^3$
$= 4 \cdot 4 \cdot 4$
$= 64$

f. $a^2 = 1.2^2$
$= (1.2)(1.2)$
$= 1.44$

PRACTICE

Evaluate the expression.

1. $5b$ when $b = 6$

2. $\frac{42}{h}$ when $h = 14$

3. $14 - b$ when $b = 11.3$

4. $v + \frac{7}{6}$ when $v = \frac{1}{3}$

5. y^4 when $y = 3$

6. q^2 when $q = 2.1$

2. Order of Operations

Vocabulary

Order of operations Established rule for evaluating an expression involving more than one operation:

Step 1: **Evaluate** expressions inside grouping symbols.

Step 2: **Evaluate** powers.

Step 3: **Multiply** and **divide** from left to right.

Step 4: **Add** and **subtract** from left to right.

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BENCHMARK 1

(Chapters 1 and 2)

EXAMPLE **Evaluate the expression.**

a. $3 \cdot 2^4 - 5 \cdot 6$ **b.** $4(3^2 + 5)$ **c.** $5[12 - (4 + 5)]$

Solution:

The multiplication that could be written in two steps ($3 \cdot 16$ evaluated first, followed by $5 \cdot 6$) is combined as one step.

a. $3 \cdot 2^4 - 5 \cdot 6 = 3 \cdot 16 - 5 \cdot 6$ **Evaluate power.**

$= 48 - 30$ **Multiply.**

$= 18$ **Subtract.**

b. $4(3^2 + 5) = 4(9 + 5)$ **Evaluate power.**

$= 4(14)$ **Add within parentheses.**

$= 56$ **Multiply.**

c. $5[12 - (4 + 5)] = 5(12 - 9)$ **Add within parentheses.**

$= 5(3)$ **Subtract within brackets.**

$= 15$ **Multiply.**

PRACTICE **Evaluate the expression.**

7. $4(10 - 3) - 5 \cdot 2$ **8.** $21 + (3^2 - 4)$ **9.** $2[42 \div (9 - 3)]$

3. Write Expressions

EXAMPLE **Translate verbal phrases into expressions.**

Keep a glossary of terms that describe each of the four basic operations.

a. The product of 8 and *m* increased by 5

b. The quotient of 8 and the difference of a number *x* and 2

c. The sum of 20 and the square of a number *n*

Solution:

a. $8m + 5$ **b.** $\frac{8}{x - 2}$ **c.** $20 + n^2$

PRACTICE **Translate the verbal phrases into expressions.**

10. The quotient when the quantity of a number *y* increased by 4 is divided by 6

11. 4 less than twice the square of a number *q*

12. 8 more than the product of a number *w* and 6

4. Write Equations and Inequalities

Vocabulary

Open sentence A mathematical statement that contains two expressions and a symbol that compares them.

Equation An open sentence that contains the symbol =.

Inequality An open sentence that contains one of the symbols <, ≤, >, or ≥.

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BENCHMARK 1

(Chapters 1 and 2)

BENCHMARK 1
A. Expressions and Equations

EXAMPLE

"No less than" (≥) and "no greater than" (≤) are opposites of "less than" (<) and "greater than" (>), respectively.

Write an equation or an inequality.

a. The difference of a number p and 12 is at most 15.

b. The product of 5 and a number m is 14.

c. A number x is at least 6 and less than 9.

Solution:

a. $p - 12 \leq 15$ **b.** $5m = 14$ **c.** $6 \leq x < 9$

PRACTICE

Write an equation or inequality.

13. The quotient of 12 and a number q is at most 5.

14. The sum of twice a number h and 5 is the same as 23.

15. The difference of a number w and 4 is greater than 12 and no more than 20.

Quiz

Evaluate the expression.

1. $\frac{h}{3} + \frac{1}{3}$ when $h = 5$ **2.** $\frac{64}{b^2}$ when $b = 4$ **3.** $12 - \frac{5a}{4}$ when $a = 4$

Evaluate the expression.

4. $(4^2 - 3) \cdot (2 + 3) + 1$ **5.** $4[(2^2 - 3) + 1]$ **6.** $\frac{[54 \div (6 - 3)^2]^2}{8 - 2}$

Translate the verbal phrases into expressions.

7. The product of twice the number y and 4 increased by 8

8. The difference of 6 times the square of a number x and 15

Write an equation or inequality.

9. The sum of the number b and 12 is twice the number b.

10. The product of a number q and 3 is no less than 10 and no more than 15.

Answer Key

Benchmark 1

A. Expressions, Equations, and Inequalities

1. 30 **2.** 3 **3.** 2.7 **4.** $\frac{3}{2}$ **5.** 81 **6.** 4.41

7. 18 **8.** 26 **9.** 14 **10.** $\frac{y + 4}{6}$ **11.** $2q^2 - 4$

12. $6w + 8$ **13.** $\frac{12}{q} \leq 5$ **14.** $2h + 5 = 23$

15. $12 < w - 4 \leq 20$

Quiz

1. 2 **2.** 4 **3.** 7 **4.** 66 **5.** 8 **6.** 6

7. $4(2y) + 8$ **8.** $6x^2 - 15$ **9.** $b + 12 = 2b$

10. $10 \leq 3q \leq 15$

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BENCHMARK 1

(Chapters 1 and 2)

B. Problem Solving (pp. 4–5)

One way to try solving a math problem is to use an organized strategy, or problem-solving plan. Read the problem to find what information is given and what you need to find out. Decide on the strategy you will use, and apply it to solve the problem. Finally, check that your solution makes sense.

1. Check Possible Solutions

Vocabulary

Solution of an equation or inequality A number that can be substituted for the variable in an equation or inequality to make a true statement.

EXAMPLE **Check whether the given number is a solution of the equation or inequality.**

a. $2x - 8 = -2$; 3 **b.** $\frac{x}{3} + 1 = 7$; 6 **c.** $x - 5 \leq 3$; 2

Solution:

a. $2(3) - 8 \stackrel{?}{=} -2$

$6 - 8 \stackrel{?}{=} -2$

$-2 = -2$ ✔

3 is a solution.

b. $\frac{6}{3} + 1 \stackrel{?}{=} 7$

$2 + 1 \stackrel{?}{=} 7$

$3 \neq 7$ ✘

6 is *not* a solution.

c. $2 - 5 \stackrel{?}{\leq} 3$

$-3 \leq 3$ ✔

2 is a solution.

PRACTICE **Check whether the given number is a solution of the equation or inequality.**

1. $5 + a < 10$; -4 **2.** $\frac{n - 3}{12} = 1$; 4 **3.** $\frac{r}{4} + 3 = 5$; -2

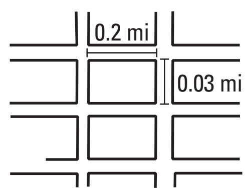
4. $-8p - 6 \leq 0$; -1 **5.** $9d - 3 = 60$; 7 **6.** $m + 8 > -7$; -14

2. Read and Understand a Problem

EXAMPLE **Read the problem below. Identify what you know and what you need to find out. You do not need to solve the problem.**

There may be more than one method that can be used to solve a problem.

You run in a city where the short blocks on north-south streets are 0.03 miles long. The long blocks on east-west streets are 0.2 mile long. You will run 2 long blocks east, a number of short blocks south, 2 long blocks west, then back to your starting point. You want to run 1.1 miles. How many short blocks should you run?



Solution:

What do you know? Each short block is 0.03 miles long. Each long block is 0.2 miles long. You will run 4 long blocks total (2 east + 2 west). You will run *s* short blocks total (south and north). You want to run a total of 2 miles.

What do you want to find out? How many short blocks should you run so that the distance you run on short blocks and the distance you run on 4 long blocks makes a total of 1.1 miles?

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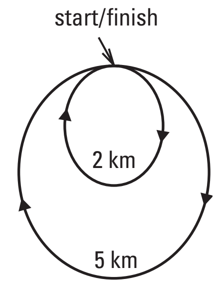
BENCHMARK 1
(Chapters 1 and 2)

PRACTICE

7. Read the problem below. Identify what you know and what you need to find out. You do not need to solve the problem.

A bicycle park has a long trail and a short trail. The long trail is 5 km long. The short trail is 2 km long. You will ride 3 laps on the short trail and some number of laps on the long trail. You want to ride 21 km. How many laps should you ride on the long trail?



3. Make a Plan

EXAMPLE **Write a verbal model of the statement below.**

How many short blocks should you run so that the distance you run on short blocks and the distance you run on 4 long blocks makes a total of 1.1 miles?

Solution:

Distance run on short blocks (miles)			+	**Distance run on long blocks** (miles)			=	**Total distance** (miles)
Length of a short block (miles/block)	•	**Number of short blocks** (block)	+	**Length of a long block** (miles/block)	•	**Number of long blocks** (block)	=	**Total distance** (miles)

PRACTICE

8. Write a verbal model for the problem in Exercise 7.

Quiz

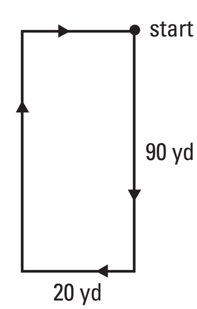
Check whether the given number is a solution of the equation or inequality.

1. $6 + j < 4; -1$ **2.** $\frac{n+5}{2} = 6; 7$ **3.** $\frac{m}{3} - 8 = -5; 9$

4. $-2y + 3 \le 0; 2$ **5.** $4g - 5 = 35; 10$ **6.** $b + 12 > 0; -13$

Read the problem below. Identify what you know and what you need to find out. Then, write a verbal model of the problem. You do not need to solve the problem.

7. Jim's grandmother exercises by walking the main rectangular hall of a local shopping mall. She walks 90 yards down the length of the hall, turns right, and walks 20 yards across the width of the hall. Then, she turns right and walks up the length of the hall again. Finally, she turns right one more time, and walks 20 yards across the width of the hall and ends up at her starting point. Jim's grandmother wants to walk 970 yards. She will walk the length of the hall 9 times. How many times will she walk across the width of the hall?



Answer Key

Benchmark 1

B. Problem Solving

1. Yes **2.** No **3.** No **4.** No **5.** Yes **6.** Yes

7. *You know:* The short trail is 2 km long and the long trail is 5 km long. You ride 3 laps on the short trail and x laps on the long. The total distance you ride is 21 km. *You want to find out:* How many laps on the long trail do you need to ride a total of 21 km?

8. Number of laps on the short trail • Length of the short trail (km) + Number of laps on the long trail • Length of the long trail (km) = Total distance (km)

Quiz

1. No **2.** Yes **3.** Yes **4.** Yes **5.** Yes **6.** No

7. *You know:* Jim's grandmother walks down the hall 90 yards, across 20 yards, up the hall 90 yards, and across 20 yards. She walks the length of the hall 9 times and she walks the width of the hall x times. She wants to walk a total of 970 yards. *You want to find out:* How many times does she need to walk the width of the hall to walk a total of 970 yards. *Verbal model:* Number of times she walks the length of the hall • Length of the hall (yd) + Number of times she walks the width of the hall • Width of the hall (yd) = Total distance (yd)

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BENCHMARK 1
(Chapters 1 and 2)

D. Operations (pp. 10–14)

Whole numbers, integers, and rational numbers are part of the set of real numbers. The following examples describe different operations with real numbers.

1. Find Opposites of Real Numbers

Vocabulary

Opposite of a real number *a* $-a$ (read "the opposite of a") is the same distance from 0 on a number line as a, but it is on the opposite side of 0.

EXAMPLE **For the given value of *a*, find $-a$.**

a. $a = 3$ **b.** $a = 7\frac{3}{5}$ **c.** $a = -5.4$

Solution:

Note that $-a$ is postive when a is negative.

a. $-a = -(3)$
$-a = -3$

b. $-a = -\left(7\frac{3}{5}\right)$
$-a = -7\frac{3}{5}$

c. $-a = -(-5.4)$
$-a = 5.4$

PRACTICE **For the given value of the variable, find the opposite.**

1. $x = -6.2$ **2.** $u = 809$ **3.** $m = 0.25$

4. $w = \frac{45}{8}$ **5.** $k = -\frac{6}{11}$ **6.** $c = -8\frac{1}{7}$

2. Find Absolute Values of Real Numbers

Vocabulary

Absolute value of a real number *a* $|a|$ (read "the absolute value of a") is the distance between a and 0 on a number line. If a is greater than or equal to 0, $|a|$ is a. If a is less than zero, $|a|$ is the opposite of a.

EXAMPLE **For the given value of *a*, find $|a|$.**

The absolute value of a number is *always* positive.

a. $a = 8$ **b.** $a = -\frac{4}{9}$ **c.** $a = 11.5$

Solution:

a. $|a| = |8| = 8$ **b.** $|a| = \left|-\frac{4}{9}\right| = -\left(-\frac{4}{9}\right) = \frac{4}{9}$

c. $|a| = |11.5| = 11.5$

PRACTICE **For the given value of the variable, find the absolute value.**

7. $b = 0.4$ **8.** $y = -50$ **9.** $p = -1.6$

10. $v = -9$ **11.** $n = \frac{23}{5}$ **12.** $h = 10\frac{8}{9}$

3. Add Real Numbers

Vocabulary

Sum The result of adding two or more real numbers.

To use a number line to find the sum of $a + b$:

- Start at a.
- If $b > 0$, you will move to the right. If $b < 0$, you will move to the left.
- Find $|b|$ and move that many units.
- The number you stop on is the sum.

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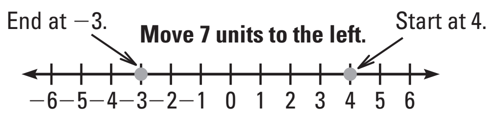
BENCHMARK 1
(Chapters 1 and 2)

EXAMPLE **Use a number line to find the sum.**

a. $4 + (-7)$ **b.** $-2 + 3$

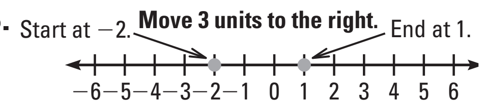
Solution:

a.



$4 + (-7) = -3$

b.



$-2 + 3 = 1$

PRACTICE **Use a number line to find the sum.**

13. $-1 + (-4)$ **14.** $2 + (-8)$ **15.** $-5 + 9$

16. $-7 + 1$ **17.** $0 + (-2)$ **18.** $-3 + (-6)$

EXAMPLE **Use the rules of real number addition to find the sum.**

a. $-19 + (-21)$ **b.** $-\frac{1}{2} + \frac{3}{2}$ **c.** $-2.8 + 1.5$

Solution:

You can find the sum of three or more numbers together by first adding two of the numbers and then adding the result to the third.

If two numbers have the same sign, add their absolute values. The sum has the same sign as the numbers added.

a. $-19 + (-21) = -(|19| + |21|) = -(19 + 21) = -40$

If two numbers have different signs, subtract the absolute value of the smaller number from the absolute value of the larger number. The sum has the same sign as the number with the larger absolute value.

b. $-\frac{1}{2} + \frac{3}{2} = \left|\frac{3}{2}\right| - \left|-\frac{1}{2}\right| = \frac{3}{2} - \frac{1}{2} = 1$

c. $-2.8 + 1.5 = -(|-2.8| - |1.5|) = -(2.8 - 1.5) = -1.3$

PRACTICE **Use the rules of real number addition to find the sum.**

19. $6.4 + (-0.3)$ **20.** $8 + (-10)$ **21.** $-100 + (-34)$

22. $s + 2\frac{2}{3} + 4\frac{3}{4}$ **23.** $-16 + 5\frac{1}{4}$ **24.** $55.1 + (-47.7)$

4. Subtract Real Numbers

Vocabulary **Difference** The result of subtracting one real number from another real number.

EXAMPLE **Find the difference.**

Use grouping symbols around negative numbers in your work to keep track of signs as you simplify expressions.

a. $5 - 18$ **b.** $-6 - 9$ **c.** $-8 - (-3)$

Solution:

To subtract b from a, add a and the opposite of b.

a. $5 - 18 = 5 + (-18) = -13$ **b.** $-6 - 9 = -6 + (-9) = -15$ **c.** $-8 - (-3) = -8 + 3 = -5$

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BENCHMARK 1
D. Operations

BENCHMARK 1

(Chapters 1 and 2)

PRACTICE **Find the difference.**

25. $-2.8 - 0.7$ **26.** $1.9 - 21.1$ **27.** $-34 - 57$

28. $3\frac{3}{5} - (-4)$ **29.** $-\frac{24}{5} - \left(-\frac{16}{15}\right)$ **30.** $73 - (-82)$

5. Multiply Real Numbers

Vocabulary **Product** The result of multiplying two or more real numbers.

EXAMPLE **Find the product.**

a. $-1.7(4)$ **b.** $-\frac{4}{5}(-10)$ **c.** $2(-3)(-8)$

To multiply three or more real numbers, first multiply two of the numbers, then multiply the result with the third number.

Solution:

The product of two numbers with the same sign is positive and the product of two numbers with different signs is negative.

a. $-1.7(4) = -6.8$

b. $-\frac{4}{5}(-10) = \frac{40}{5}$
$= 8$

c. $2(-3)(-8) = [2(-3)](-8)$
$= -6(-8)$
$= 48$

PRACTICE **Find the product.**

31. $10(-3)$ **32.** $-\frac{11}{4}(6)$ **33.** $-\frac{5}{8}\left(-\frac{24}{15}\right)\left(-\frac{25}{47}\right)$

34. $-12(4)(-3)$ **35.** $2.5(10.4)(-7)$ **36.** $-2.4(-9.1)$

Vocabulary **Multiplicative inverse of a real number *a*** The reciprocal of a, or $\frac{1}{a}$. The product of a and its multiplicative inverse is 1.

EXAMPLE **Find the multiplicative inverse of *a*.**

You can check your answer by multiplying the original number by its inverse and making sure the product is 1.

a. $a = 9$ **b.** $a = -4$ **c.** $a = -\frac{1}{8}$

Solution:

a. $\frac{1}{a} = \frac{1}{9}$ **b.** $\frac{1}{a} = \frac{1}{-4} = -\frac{1}{4}$ **c.** $\frac{1}{a} = \frac{1}{-\frac{1}{8}} = -\frac{8}{1} = -8$

PRACTICE **Find the multiplicative inverse of the number.**

37. -6 **38.** 1 **39.** $\frac{3}{4}$

40. $-\frac{7}{5}$ **41.** $9\frac{1}{2}$ **42.** $-5\frac{15}{32}$

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BENCHMARK 1
(Chapters 1 and 2)

6. Divide Real Numbers

Vocabulary **Quotient** The result dividing a real number by another real number.

EXAMPLE **Find the quotient.**

Division by 0 is undefined, because 0 does not have a multiplicative inverse.

a. $35 \div (-7)$ **b.** $-26 \div (-13)$ **c.** $\frac{2}{3} \div \left(-\frac{12}{18}\right)$

Solution:

a. $35 \div (-7) = 35 \cdot \left(-\frac{1}{7}\right)$
$= -\frac{35}{7}$
$= -5$

b. $-26 \div (-13) = -26 \cdot \left(-\frac{1}{13}\right)$
$= \frac{26}{13}$
$= 2$

c. $\frac{2}{3} \div \left(-\frac{12}{18}\right) = \frac{2}{3} \cdot \left(-\frac{18}{12}\right)$
$= -\frac{36}{36}$
$= -1$

PRACTICE **Find the quotient.**

43. $-92 \div (-4)$ **44.** $-2\frac{1}{4} \div \frac{5}{8}$ **45.** $9 \div \frac{1}{9}$

46. $1 \div \left(-\frac{5}{2}\right)$ **47.** $-\frac{32}{15} \div (-8)$ **48.** $-6\frac{2}{3} \div 10\frac{4}{9}$

7. Find Square Roots

Vocabulary **Square root of *a*** If $b^2 = a$, then b is the square root of a. Every positive nonzero real number a has two square roots, $-\sqrt{a}$ and $\sqrt{a}$.

Radicand The number or expression inside a radical symbol.

EXAMPLE **Evaluate the expression.**

The symbol ± in front of a number refers to the number and its opposite. For example, "±6" is the same as "6 and −6."

a. $\pm\sqrt{49}$ **b.** $\sqrt{1}$ **c.** $-\sqrt{144}$

Solution:

a. ± 7 **b.** 1 **c.** -12

PRACTICE **Evaluate the expression.**

49. $-\sqrt{400}$ **50.** $\pm\sqrt{9}$ **51.** $\sqrt{81}$

52. $\sqrt{0}$ **53.** $\sqrt{4}$ **54.** $\pm\sqrt{900}$

BENCHMARK 1
(Chapters 1 and 2)

Quiz

For the given value of the variable, find the opposite, absolute value, and multiplicative inverse.

1. $a = -16$ **2.** $y = 7\frac{3}{10}$ **3.** $r = -0.3$

Evaluate the expression.

4. $51 - (-65)$ **5.** $-\frac{5}{7} \cdot \frac{21}{40}$ **6.** $6\frac{5}{9} \div \left(-1\frac{2}{3}\right)$

7. $8 + (-15)$ **8.** $-2(-35)$ **9.** $\sqrt{64}$

10. $-18 \div \frac{12}{5}$ **11.** $\pm\sqrt{81}$ **12.** $-2.3 - 4.9$

Answer Key

Benchmark 1

D. Operations

1. 6.2 **2.** -809 **3.** -0.25 **4.** $-\frac{45}{8}$ **5.** $\frac{6}{11}$

6. $8\frac{1}{7}$ **7.** 0.4 **8.** 50 **9.** 1.6 **10.** 9 **11.** $\frac{23}{5}$

12. $10\frac{8}{9}$ **13.** -5 **14.** -6 **15.** 4 **16.** -6

17. -2 **18.** -9 **19.** 6.1 **20.** -2 **21.** -134

22. $7\frac{5}{12}$ **23.** $-10\frac{3}{4}$ **24.** 7.4 **25.** -3.5

26. -19.2 **27.** -91 **28.** $7\frac{3}{5}$ **29.** $-\frac{56}{15}$

30. 155 **31.** -30 **32.** $-\frac{33}{2}$ **33.** $-\frac{25}{47}$

34. 144 **35.** -182 **36.** 21.84 **37.** $-\frac{1}{6}$

38. 1 **39.** $\frac{4}{3}$ **40.** $-\frac{5}{7}$ **41.** $\frac{2}{19}$ **42.** $-\frac{32}{175}$

43. 23 **44.** $-\frac{18}{5}$ **45.** 81 **46.** $-\frac{2}{5}$ **47.** $\frac{4}{15}$

48. $-\frac{30}{47}$ **49.** -20 **50.** ± 3 **51.** 9 **52.** 0

53. 2 **54.** ± 30

Quiz

1. 16; 16; $-\frac{1}{16}$ **2.** $-7\frac{3}{10}$; $7\frac{3}{10}$; $\frac{10}{73}$

3. 0.3; 0.3; $-\frac{10}{3}$ **4.** 116 **5.** $-\frac{3}{8}$ **6.** $-\frac{59}{15}$

7. -7 **8.** 70 **9.** 8 **10.** $-\frac{15}{2}$ **11.** ± 9

12. -7.2

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BENCHMARK 2

(Chapters 3 and 4)

A. Solving Equations in one Variable (pp. 19–22)

To solve an equation in one variable, isolate the variable on one side of the equation. The following examples illustrate different ways to isolate the variable.

1. Solve an Equation Using Addition or Subtraction

Vocabulary

Inverse operations Two operations that undo each other, such as addition and subtraction or multiplication and division.

Equivalent equations Equations that have the same solution(s).

EXAMPLE **Use addition or subtraction to solve the equation.**

a. $x + 9 = 3$ **b.** $x - 5 = 2$ **c.** $x + 4.1 = 6$

Solution:

Be sure to subtract (or add) the same number from each side, so that the new equation is *equivalent* to the original equation.

a. $x + 9 = 3$ **Write original equation.**

$x + 9 - 9 = 3 - 9$ **Use subtraction property of equality. Subtract 9 from each side.**

$x = -6$ **Simplify.**

The solution is −6.

b. $x - 5 = 2$ **Write original equation.**

$x - 5 + 5 = 2 + 5$ **Use addition property of equality. Add 5 to each side.**

$x = 7$ **Simplify.**

The solution is 7.

c. $x + 4.1 = 6$ **Write original equation.**

$x + 4.1 - 4.1 = 6 - 4.1$ **Subtract 4.1 from each side.**

$x = 1.9$ **Simplify.**

The solution is 1.9.

PRACTICE **Use addition or subtraction to solve the equation.**

1. $x + 5 = 4$ **2.** $c - 3 = 8$ **3.** $t + 6 = 10$

2. Solve an Equation Using Multiplication or Division

EXAMPLE **Use multiplication or division to solve the equation.**

a. $\frac{x}{6} = 3$ **b.** $-7x = -49$ **c.** $-\frac{3}{8}x = 5$

Solution:

a. $\frac{x}{6} = 3$ **Write original equation.**

$6 \cdot \frac{x}{6} = 6 \cdot 3$ **Multiply each side by 6.**

$x = 18$ **Simplify.**

The solution is 18.

BENCHMARK 2 A. Solving Equations

BENCHMARK 2
(Chapters 3 and 4)

b. $-7x = -49$ **Write original equation.**

$\frac{-7x}{-7} = \frac{-49}{-7}$ **Divide each side by −7.**

$x = 7$ **Simplify.**

The solution is 7.

c. $-\frac{3}{8}x = 5$ **Write original equation.**

Recall that the product of a number and its *reciprocal* is 1.

$-\frac{8}{3}\left(-\frac{3}{8}x\right) = -\frac{8}{3}(5)$ **Multiply each side by the reciprocal $-\frac{8}{3}$.**

$x = -\frac{40}{3}$ **Simplify.**

The solution is $-\frac{40}{3}$.

PRACTICE **Use multiplication or division to solve the equation.**

4. $\frac{r}{10} = 2$ **5.** $4q = 32$ **6.** $\frac{a}{9} = -3$

3. Solve a Two-Step Equation

Vocabulary **Order of operations** The rules for evaluating an expression involving more than one operation.

EXAMPLE **Solve the equation.**

a. $7x + 1 = 4$ **b.** $\frac{x}{5} - 10 = 20$

Solution:

a. $7x + 1 = 4$ **Write original equation.**

$7x + 1 - 1 = 4 - 1$ **Subtract 1 from each side.**

$7x = 3$ **Simplify.**

$\frac{7x}{7} = \frac{3}{7}$ **Divide each side by 7.**

$x = \frac{3}{7}$ **Simplify.**

The solution is $\frac{3}{7}$.

b. $\frac{x}{5} - 10 = 20$ **Write original equation.**

$\frac{x}{5} - 10 + 10 = 20 + 10$ **Add 10 to each side.**

$\frac{x}{5} = 30$ **Simplify.**

$5 \cdot \frac{x}{5} = 5 \cdot 30$ **Multiply each side by 5.**

$x = 150$ **Simplify.**

The solution is 150.

PRACTICE **Solve the equation.**

7. $3 + 4x = 11$ **8.** $7.5a - 10 = -32.5$ **9.** $\frac{t}{8} + 6 = 3$

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BENCHMARK 2

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4. Solve Multi-Step Equations

EXAMPLE **Solve $4x + 3(x - 5) = -12$.**

Solution:

Distributive Property: $a(b + c) = ab + ac$, $a(b - c) = ab - ac$

$4x + 3(x - 5) = -12$	**Write original equation.**
$4x + 3x - 15 = -12$	**Eliminate the parentheses by using the distributive property.**
$7x - 15 = -12$	**Combine like terms.**
$7x - 15 + 15 = -12 + 15$	**Add 15 to each side.**
$7x = 3$	**Simplify.**
$\frac{7x}{7} = \frac{3}{7}$	**Divide each side by 7.**
$x = \frac{3}{7}$	**Simplify.**

The solution is $\frac{3}{7}$.

PRACTICE **Solve the equation.**

10. $-2(4a + 5) - 6a = 10$ **11.** $6 + 3(n - 7) = 12$ **12.** $\frac{2}{3}(6g + 1) + \frac{1}{3} = -19$

5. Solve Equations with Variables on Both Sides

EXAMPLE **Solve $x - 4 = 3x + 8$.**

Solution:

You could also begin solving the equation by adding 4 to each side to obtain $x = 3x + 12$. You will get the same solution when you finish solving for x.

$x - 4 = 3x + 8$	**Write original equation.**
$x - 4 - 3x = 3x + 8 - 3x$	**Subtract $3x$ from each side.**
$-2x - 4 = 8$	**Simplify each side.**
$-2x - 4 + 4 = 8 + 4$	**Add 4 to each side.**
$-2x = 12$	**Simplify.**
$\frac{-2x}{-2} = \frac{12}{-2}$	**Divide each side by -2.**
$x = -6$	**Simplify.**

The solution is -6.

PRACTICE **Solve the equation.**

13. $8t - 10 = 5 + 3t$ **14.** $-9 - 4h = -2h + 3$ **15.** $12d + 4 = 6 - d$

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BENCHMARK 2

(Chapters 3 and 4)

6. Identify the Number of Solutions to an Equation

Vocabulary **Identity** An equation that is true for all values of the variable.

EXAMPLE **Solve the equation, if possible.**

a. $-8x = -4(2x + 1)$ **b.** $-5x - 15 = -5(x + 3)$

Solution:

An equation can have *one* solution, *no* solution, or *all real numbers* as solutions.

a.

$-8x = -4(2x + 1)$	**Write original equation.**
$-8x = -8x - 4$	**Distributive property**
$-8x + 8x = -8x - 4 + 8x$	**Add 8x to each side.**
$0 = -4$ ✘	**Simplify.**

The statement $0 = -4$ is not true, so the equation has no solution.

b.

$-5x - 15 = -5(x + 3)$	**Write original equation.**
$-5x - 15 = -5x - 15$	**Distributive property**

The statement $-5x - 15 = -5x - 15$ is true for all values of x. So the equation is an identity, and the solution is all real numbers.

PRACTICE **Solve the equation, if possible.**

16. $7(3s - 3) = 3(7s - 7)$ **17.** $-6 + 3(v - 9) = 6v + 27$

18. $6a + 1 = 2(3a - 1)$

Quiz

Use addition or subtraction to solve the equation.

1. $w + 3.6 = 8.9$ **2.** $p - 7.2 = -5$ **3.** $v + 12 = -3$

Use multiplication or division to solve the equation.

4. $5n = -6$ **5.** $\frac{g}{2.1} = 5$ **6.** $1.2z = 8.4$

Solve the equation.

7. $9x - 2 = 0$ **8.** $-4 + \frac{b}{2.5} = 40$

9. $24 = \frac{v}{6} - 3$ **10.** $9y + 5(y - 9) = 39$

11. $-c + 4(8 - c) = -43$ **12.** $\frac{1}{4}m - \left(\frac{3}{4}m + 2\right) = 8$

13. $\frac{1}{2}n + 4 = -\frac{3}{2}n - 18$ **14.** $4.3 + 2.3r = 7.1 - 1.9r$

15. $z - 26 = 5z - 36$

Solve the equation, if possible.

16. $4(b - 5) = 5(b - 4)$ **17.** $8p - 12 = -4(-2p + 3)$

18. $5(k + 3) - k = 3k + 5$

Benchmark 2

A. Solving Equations in One Variable

1. -1 **2.** 11 **3.** 4 **4.** 20 **5.** 8 **6.** -27 **7.** 2

8. -3 **9.** -24 **10.** $-\frac{10}{7}$ **11.** 9 **12.** -5

13. 3 **14.** -6 **15.** $\frac{2}{13}$ **16.** All real numbers

17. -20 **18.** No solution

Quiz

1. 5.3 **2.** 2.2 **3.** -15 **4.** $-\frac{6}{5}$ **5.** 10.5 **6.** 7

7. $\frac{2}{9}$ **8.** 110 **9.** 162 **10.** 6 **11.** 15

12. -20 **13.** -11 **14.** $\frac{2}{3}$ **15.** $\frac{5}{2}$

16. 0 **17.** All real numbers **18.** -10

BENCHMARK 2
(Chapters 3 and 4)

B. Proportion and Percent Problems (pp. 23–26)

The comparison of two quantities by division is called a *ratio*. The following examples illustrate how to write and use ratios.

1. Write a Ratio

EXAMPLE **Kim has a jar containing 45 pennies, 18 nickels, 30 dimes, and 42 quarters. Write the specified ratio in simplest form.**

a. number of nickels to number of pennies **b.** number of quarters to number of dimes **c.** number of pennies to total number of coins

The *ratio* of two quantities *a* and *b* can be written in three ways: *a* to *b*, *a* : *b*, or $\frac{a}{b}$.

Solution:

a. $\frac{\text{nickels}}{\text{pennies}} = \frac{18}{45} = \frac{2}{5}$

b. $\frac{\text{quarters}}{\text{dimes}} = \frac{42}{30} = \frac{7}{5}$

c. $\frac{\text{pennies}}{\text{total}} = \frac{45}{45 + 18 + 30 + 42} = \frac{45}{135} = \frac{1}{3}$

PRACTICE **On his last report card, Jay earned 2 A's, 3 B's, and 1 C. Write the specified ratio in simplest form.**

1. the number of A's to the number of B's
2. the number of C's to the number of A's and B's
3. the number of B's to the total number of grades

A school band orders t-shirts. They order 12 smalls, 10 mediums, and 15 larges. Write the specified ratio in simplest form.

4. the number of smalls to the total number of t-shirts
5. the number of mediums to the number of larges
6. the number of larges and smalls to the number of mediums

2. Solve a Proportion

Vocabulary **Proportion** An equation showing that two ratios are equivalent.

EXAMPLE **Solve the proportion $\frac{2}{3} = \frac{x}{15}$.**

Solution:

$\frac{2}{3} = \frac{x}{15}$ **Write original proportion.**

$15 \cdot \frac{2}{3} = 15 \cdot \frac{x}{15}$ **Multiply each side by 15.**

$\frac{30}{3} = x$ **Simplify.**

$10 = x$ **Divide.**

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BENCHMARK 2
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PRACTICE **Solve the proportion.**

7. $\frac{7}{42} = \frac{t}{84}$ **8.** $\frac{5}{6} = \frac{k}{72}$ **9.** $\frac{a}{65} = \frac{6}{39}$

10. $\frac{v}{3} = \frac{85}{51}$ **11.** $\frac{8}{20} = \frac{n}{15}$ **12.** $\frac{q}{54} = \frac{8}{36}$

3. Use the Cross Products Property

Vocabulary **Cross product** The product of the numerator of one ratio in a proportion and the denominator of the other ratio in the proportion.

EXAMPLE **Solve the proportion $\frac{2}{5} = \frac{x}{20}$.**

Solution:

Cross Products Property: The cross products of a proportion are equal.

$\frac{2}{5} = \frac{x}{20}$ **Write original proportion.**

$2 \cdot 20 = 5 \cdot x$ **Cross products property**

$40 = 5x$ **Simplify.**

$8 = x$ **Divide each side by 5.**

PRACTICE **Use the cross products property to solve the proportion.**

13. $\frac{y}{12} = \frac{19}{4}$ **14.** $\frac{15}{35} = \frac{p}{63}$ **15.** $\frac{13}{36} = \frac{65}{w}$

16. $\frac{7}{30} = \frac{196}{m - 5}$ **17.** $\frac{5}{8} = \frac{n}{n + 9}$ **18.** $\frac{18}{v - 1} = \frac{48}{3v - 5}$

4. Find a Percent Using a Proportion

EXAMPLE **What percent of 50 is 12?**

Solution:

Represent "*a* is *p* percent of *b*" using the proportion $\frac{a}{b} = \frac{p}{100}$.

$\frac{a}{b} = \frac{p}{100}$ **Write proportion.**

$\frac{12}{50} = \frac{p}{100}$ **Substitute 12 for *a* and 50 for *b*.**

$1200 = 50p$ **Cross products property**

$24 = p$ **Divide each side by 50.**

12 is 24% of 50.

PRACTICE **Use a proportion to answer the question.**

19. What is 34% of 60? **20.** 5 is 4% of what number?

21. 16 is what percent of 20? **22.** 9 is what percent of 50?

23. What is 5% of 76? **24.** 17 is 25% of what number?

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BENCHMARK 2
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5. Use the Percent Equation

EXAMPLE **Use the percent equation to answer the question.**

a. What number is 13% of 42?

b. What percent of 15 is 13.8?

c. 84 is 60% of what number?

Solution:

Represent "*a* is *p* percent of *b*" using the equation $a = p\% \cdot b$ where *a* is a part of the base *b* and *p*% is the percent.

a. $a = p\% \cdot b$ — **Write percent equation.**

$= 13\% \cdot 42$ — **Substitute 13 for *p* and 42 for *b*.**

$= 0.13 \cdot 42$ — **Write percent as decimal.**

$= 5.46$ — **Multiply.**

5.46 is 13% of 42.

b. $a = p\% \cdot \text{b}$ — **Write percent equation.**

$13.8 = p\% \cdot 15$ — **Substitute 13.8 for *a* and 15 for *b*.**

$0.92 = p\%$ — **Divide each side by 15.**

$92\% = p\%$ — **Write percent as decimal.**

13.8 is 92% of 15.

c. $a = p\% \cdot b$ — **Write percent equation.**

$84 = 60\% \cdot b$ — **Substitute 84 for *a* and 60 for *p*.**

$84 = 0.6 \cdot b$ — **Write percent as decimal.**

$140 = b$ — **Divide each side by 0.6.**

84 is 60% of 140.

PRACTICE **Use the percent equation to answer the question.**

25. What is 73% of 2500?

26. 3 is 4% of what number?

27. 72 is what percent of 288?

28. 26 is what percent of 78?

29. What is 99% of 23?

30. 66 is 33% of what number?

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BENCHMARK 2

(Chapters 3 and 4)

Quiz

A shelter has 24 cats, 18 dogs, and 3 birds. Write each ratio in simplest form.

1. the number of dogs to the total number of animals
2. the number of dogs to the number of cats
3. the number of birds to the number of cats and dogs

Solve the proportion.

4. $\frac{48}{92} = \frac{8}{n}$
5. $\frac{84}{d} = \frac{70}{25}$
6. $\frac{20}{24} = \frac{3a}{90}$
7. $\frac{76}{d} = \frac{19}{13}$
8. $\frac{2}{7} = \frac{46}{s - 2}$
9. $\frac{18}{7x + 3} = \frac{3}{x + 2}$

Answer the question.

10. What is 38% of 38?
11. 9 is 6% of what number?
12. 51 is what percent of 60?
13. 18 is 3% of what number?
14. 78 is what percent of 120?
15. What is 51% of 9?

Answer Key

Benchmark 2

B. Proportion and Percent Problems

1. $\frac{2}{3}$ **2.** $\frac{1}{5}$ **3.** $\frac{1}{2}$ **4.** $\frac{12}{37}$ **5.** $\frac{2}{3}$ **6.** $\frac{27}{10}$ **7.** 14
8. 60 **9.** 10 **10.** 5 **11.** 6 **12.** 12 **13.** 57
14. 27 **15.** 180 **16.** 845 **17.** 15 **18.** 7
19. 20.4 **20.** 125 **21.** 80% **22.** 18% **23.** 3.8
24. 68 **25.** 1825 **26.** 75 **27.** 25%
28. 33.3% **29.** 22.77 **30.** 200

Quiz

1. $\frac{2}{5}$ **2.** $\frac{3}{4}$ **3.** $\frac{1}{14}$ **4.** $15\frac{1}{3}$ **5.** 30 **6.** 25
7. 52 **8.** 163 **9.** 9 **10.** 14.44 **11.** 150
12. 85% **13.** 600 **14.** 65% **15.** 4.59